Package Macromodeling via Time-Domain Vector Fitting

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***Abstract*—This paper addresses the construction of lumped macromodels for package structures. A technique named Time-Domain Vector Fitting (TD-VF) is introduced for the identification of the dominant poles of the structure. This method uses as raw data transient excitations and responses at the ports of the structure. These responses are easily obtained from transient full-wave electromagnetic solvers based, e.g., on Finite Differences. The rational approximation can be easily synthesized into a SPICE-compatible subcircuit providing a broadband approximation to the input-output behavior of the package.**

***Index Terms*—Circuit extraction, macromodeling, time-domain vector fitting, vector fitting.**

I. INTRODUCTION

THE design of modern high-speed electronic systems requires careful modeling for the assessment of Signal Integrity (SI) and Electromagnetic Compatibility (EMC) issues. Indeed, the ever increasing clock speed of digital systems requires consideration of parasitic effects that were not significant at lower operation frequencies. As an example, it is well known that electrical interconnects at chip, multichip, package, and board level are one of the most critical parts for SI and EMC due to mutual couplings, transmission-line effects, frequency-dependent conductor losses, etc. Accurate modeling of such structures can be achieved by full-wave electromagnetic solvers, which are however too complex for global system-level simulations.

Macromodeling techniques allow us to tackle the system-level modeling problem by splitting the entire system into subparts interacting with each other only through well-defined ports. Each part is characterized separately using specific tools, with the aim of generating an equivalent circuit providing an approximation of its input/output port behavior. This equivalent circuit is used for subsequent system-level simulation within a standard circuit analysis environment. At this stage, inclusion of nonlinear/dynamic terminations like drivers and receivers can be done easily.

Several macromodeling techniques are available in the literature (see, e.g., [2], [3], [5], [7]). Each of these techniques is tailored to the specific form in which the structure under investigation is characterized. We focus here on linear macromodeling from transient port responses, and we present a new macromodeling technique that we denote Time-Domain Vector Fitting (TD-VF). The algorithm is an extension of the well-known standard Vector-Fitting (VF) technique [5], whose accuracy and efficiency is widely recognized. Standard VF operates entirely in frequency domain, providing rational approximations of transfer functions starting from frequency-domain samples. Conversely, the proposed TD-VF technique produces rational approximations directly from transient input/output responses. A specific application where this approach is convenient is the equivalent circuit extraction for three-dimensional interconnect structures (like, e.g., electronic packages or connectors), that are characterized using a full-wave electromagnetic solver based, e.g., on the Finite-Difference Time-Domain (FDTD) method [8]. Due to the high computational cost, it is desirable to terminate the full-wave analysis before all transients have extinguished, thus obtaining truncated time responses. The presented algorithm is ideally suited for this type of native data, whereas a frequency-domain approach would not be feasible. Section II presents the formulation of the TD-VF algorithm. Numerical examples are presented in Section III.

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II. TIME-DOMAIN VECTOR FITTING

We consider a package structure with an arbitrary number P of ports. Usually, a transient characterization of such a multiport structure is obtained by exciting one port at the time and computing/measuring the responses at all ports. As a result, the raw dataset for the package characterization is a matrix of response waveforms $y_{ij}(t)$ at port (i), due to excitation $x_j(t)$ at port (j). We remark that this type of dataset is the natural outcome of time-domain full-wave electromagnetic solvers. The objective is the derivation of a rational approximation to the matrix transfer function

$$\mathbf{H}(s) \simeq \mathbf{H}_\infty + \sum_{n=1}^{N} \frac{\mathbf{R}_n}{s - p_n}. \tag{1}$$

Note that the N poles are common to all matrix entries, whereas the direct coupling term and the residues are $P \times P$ matrices. The raw transient responses satisfy the relation

$$y_{ij}(t) = \mathcal{L}^{-1}\{H_{ij}(s)\} * x_j(t), \quad i,j = 1,\ldots,P \tag{2}$$

where $\mathcal{L}^{-1}$ is the inverse Laplace operator and $*$ denotes convolution.

The TD-VF algorithm is a time-domain formulation of the well-known standard Vector Fitting algorithm [5]. It is based on a combination of digital filtering and linear least squares approximations. A first step allows the identification of the dominant poles of the structure. Once these poles are known, the matrices of residues are computed. Finally, a state-space realization is derived, leading to a direct equivalent circuit synthesis. These three steps are detailed below.

A. Poles Identification

We introduce, as for standard VF [5], a scalar weight function

$$\sigma(s) = 1 + \sum_{n=1}^{N} \frac{k_n}{s - q_n} = \frac{\prod_{n=1}^{N}(s - z_n)}{\prod_{n=1}^{N}(s - q_n)} \quad (3)$$

with known (initial) poles $\{q_n\}$ and unknown residues $\{k_n\}$. Let us assume now that the following approximation holds:

$$\sigma(s)\mathbf{H}(s) \simeq \mathbf{M}_\infty + \sum_{n=1}^{N} \frac{\mathbf{M}_n}{s - q_n}. \quad (4)$$

Since the right-hand-side has the same poles $\{q_n\}$ as the weight function, a cancellation between the zeros $\{z_n\}$ of $\sigma(s)$ and the poles $\{p_n\}$ of $\mathbf{H}(s)$ must occur. This condition provides indeed a way to estimate these poles by solving (4) for the unknown residues $\{k_n\}$, computing the zeros $\{z_n\}$ using standard techniques [5], and by enforcing $p_n = z_n$. This procedure, named *pole relocation*, avoids use of ill-conditioned nonlinear least squares algorithms for the direct approximation of (1). Also, the poles relocation can be iterated using the estimated poles as starting poles for the new iteration. Convergence is usually reached in few iterations, as discussed in Section III.

Standard VF solves (4) via linear least squares using the available raw data $\mathbf{H}(s)$ at given frequency points $s = j\omega$. Instead, we reformulate this condition in time domain by applying (4) to the vector of input signals $\mathbf{X}(s)$ and applying inverse Laplace transform. The resulting condition reads componentwise:

$$y_{ij}(t) \simeq M_{\infty,ij}x_j(t) + \sum_{n=1}^{N} M_{n,ij}x_{n,j}(t) - \sum_{n=1}^{N} k_n y_{n,ij}(t), \quad i,j = 1,\ldots,P \quad (5)$$

where the transient waveforms

$$x_{n,j}(t) = \int_0^t e^{q_n(t-\tau)}x_j(\tau)d\tau, \quad (6)$$

$$y_{n,ij}(t) = \int_0^t e^{q_n(t-\tau)}y_{ij}(\tau)d\tau \quad (7)$$

are convolutions resulting from inverse Laplace transform of each partial fraction in the expansions (3)–(4). These waveforms are easily obtained by applying a suitable discretization of the convolution integrals. We use here a linear interpolation between the raw samples, leading to

$$x_{n,j}(t_{k+1}) = \alpha_n x_{n,j}(t_k) + \beta_n^0 x_j(t_{k+1}) + \beta_n^1 x_j(t_k) \quad (8)$$

corresponding to a first-order IIR filter with weights

$$\alpha_n = e^{q_n\Delta t}, \quad \beta_n^0 = \frac{-1 - q_n\Delta t + e^{q_n\Delta t}}{q_n^2\Delta t}, \quad \beta_n^1 = \frac{1 + (q_n\Delta t - 1)e^{q_n\Delta t}}{q_n^2\Delta t}. \quad (9)$$

A similar expression holds for the output responses $y_{n,ij}(t)$.

Condition (5) is enforced in least squares sense using raw and filtered input/output sequences. The residues k_n of the weight function are common in all the P^2 independent equations in (5), which result coupled and not independent. These residues are the only quantities that are needed for further processing.

Equation (5) is presented assuming that all available output responses at all ports are used for the poles estimation. However, sufficiently accurate poles estimates can often be obtained by processing a significant subset of (dominant) port responses, e.g., only diagonal entries with $i = j$. This is often the case for packages exploiting weak coupling between nonadjacent pins. In such case, the overall system results much smaller. An example will be provided in Section III . We remark that the raw and filtered waveforms can be sub-sampled in order to reduce the size of the size of the system (5). In fact, the subsampling rate can be related to the effective bandwidth of the excitation waveform (usually a Gaussian) via the Nyquist rule. As a result, for typical package structures, the resulting equivalent samples count per signal is about 50–100. Note also that the sparse structure of the system matrix in (5) can be exploited to optimize the performance of the algorithm.

B. Residues

Once the poles $\{p_n\}$ are known, the residues $\mathbf{R}_n$ and the direct coupling matrix $\mathbf{H}_\infty$ in (1) are computed componentwise:

$$y_{ij}(t) \simeq H_{\infty,ij}x_j(t) + \sum_{n=1}^{N} R_{n,ij}\widehat{x}_{n,j}(t) \quad (10)$$

with $\widehat{x}_{n,j}(t)$ defined as in (6) with $\{q_n\}$ replaced by the estimated poles $\{p_n\}$.

C. State-Space Realization

The final step in the algorithm is the identification of a state-space realization

$$\begin{cases} \frac{d}{dt}\mathbf{w}(t) = \mathbf{A}\mathbf{w}(t) + \mathbf{B}\mathbf{x}(t) \\ \mathbf{y}(t) = \mathbf{C}\mathbf{w}(t) + \mathbf{D}\mathbf{x}(t) \end{cases} \quad (11)$$

from the poles and residues representation in (1) Once the state matrices are found, they can be used to synthesize SPICE-like equivalent circuits through well known techniques. The particular form of the state matrices depends on some arbitrary choice, since several realizations are possible. We can use one of the simplest possibilities, namely a realization in Jordan canonical form. This amounts to setting $\mathbf{A} = \mathrm{diag}\{p_n\}$. Therefore

$$\mathbf{H}(s) \simeq \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D} = \sum_{n=1}^{N} \frac{\mathbf{c}_n\mathbf{b}_n^T}{s - p_n} + \mathbf{D} \quad (12)$$

where $\mathbf{c}_n$ and $\mathbf{b}_n^T$ are the columns of $\mathbf{C}$ and the rows of $\mathbf{B}$, respectively. This expression indicates that residues $\mathbf{H}_n = \mathbf{c}_n\mathbf{b}_n^T$ have a theoretical unitary rank. However, since the actual estimates come from a least squares fit, the numerical rank will be generally larger than one. Therefore, some matrix factorization can be applied to $\mathbf{H}_n$ to find suitable minimal realizations that retain the accuracy of the performed fit. In particular, we adopt here the procedure in [1], which is based on the singular value decomposition. The direct coupling matrix is $\mathbf{D} = \mathbf{H}_\infty$.

A final remark on passivity. It is well-known that only strictly passive macromodels can be effectively used, since stable but

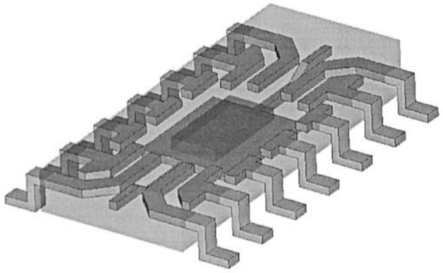

Fig. 1. Package structure used for illustration of the TD-VF algorithm. Bondwires (not shown in the picture) were also included in the model.

nonpassive circuits may lead to instabilities depending on the termination networks. The proposed TD-VF algorithm is not passive by construction, as it provides an approximation to the input-output responses of the structure. Therefore, the passivity of the generated macromodel will depend both on the passivity of the raw transient data and on the accuracy of the rational approximation algorithm. The former condition is usually satisfied if a stable field solver is used for the generation of the raw data. We will show through the examples in Section III that the level of accuracy of TD-VF is excellent. Therefore, if some passivity test is applied to the macromodel and if some passivity violation occurs, this violation will be small. In this case, some standard passivity compensation can be applied without difficulties. See [4] and [6] for details.

III. Numerical Examples

In order to test the TD-VF algorithm, a commercial 14-pin surface mount package has been meshed and analyzed through a full-wave electromagnetic solver based on the Finite-Difference Time-Domain method [8]. The structure, depicted in Fig. 1, has 28 ports, since each pin leads to one port on the board side (odd-numbered) and one port on the die side (even-numbered). As a result, a complete set of 28×28 responses (transient scattering waves with reference load $R_0 = 50\ \Omega$) have been obtained, using a unitary Gaussian pulse as excitation, with a 20 dB bandwidth of 30 GHz. Only the selected responses $\{y_{jj}, y_{j+1,j}, y_{j+2,j}\}$, i.e., reflected and transmitted wave on each excited pin, plus near-end crosstalk, were used for the determination of the dominant poles, with dynamic order N ranging from 11 up to 30. Fig. 2 depicts the maximum approximation error among all responses. Although no theoretical relation between maximum approximation error and order of the macromodel can be argued, the figure shows that the rational approximation is convergent with an increasing order. There is a clear tradeoff between macromodel complexity and accuracy. The macromodel order should therefore be determined based on the accuracy requirements for the specific application it is being derived for. As an example, Fig. 3 shows a comparison between some transient scattering responses of the original FDTD simulation and the corresponding responses of the macromodel of order $N = 20$. No difference can be seen on this scale between corresponding waveforms. This example illustrates that the proposed TD-VF technique leads to very accurate macromodels of structures with a possibly large number

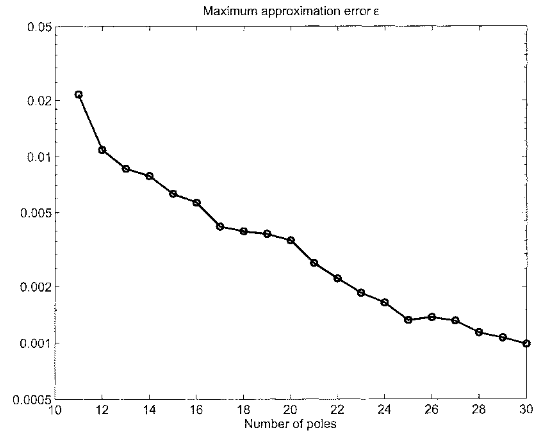

Fig. 2. Macromodeling of a commercial 14-pin package. Maximum approximation error among all 28×28 port responses for various approximation orders N.

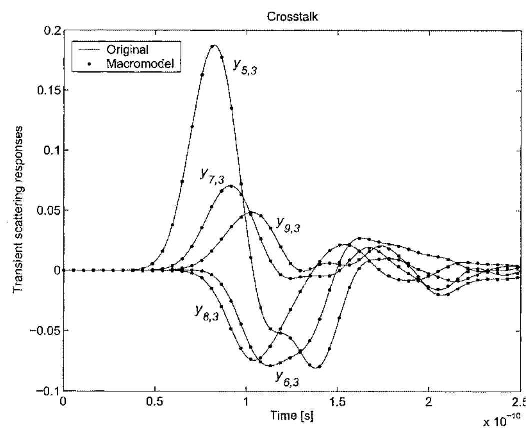

Fig. 3. Macromodeling of a commercial 14-pin package. Comparison between original and macromodel responses for selected transient scattering waveforms.

of ports. The core of the algorithm is very straightforward and simple, yet robust and reliable. Further application examples are being developed and will be published elsewhere.

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